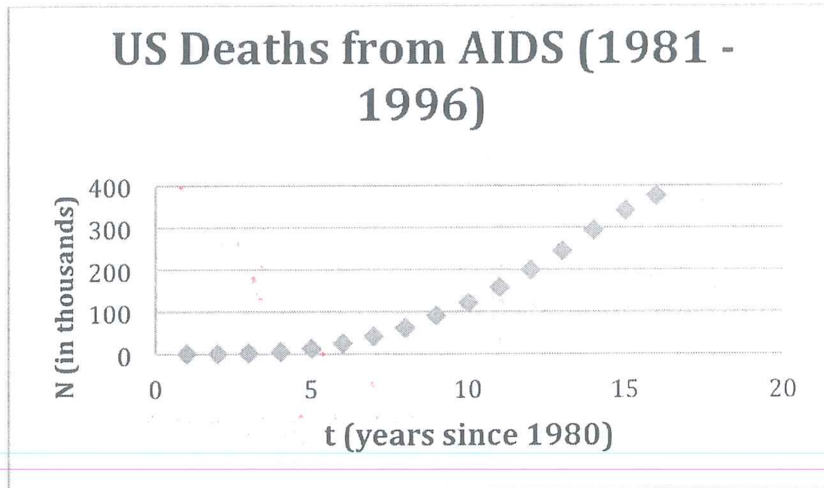
Sec. 11.7 Fitting Exponentials and Polynomials to Data

Visualizing the Data

The data in the table gives the total number of deaths in the US from AIDS from 1981 to 1996. The graph suggests that a linear function may not give the best possible fit for these data.

t	N
1	159
2	622
3	2130
4	5635
5	12607
6	24717
7	41129
8	62248
9	90039
10	121577
11	158193
12	199287
13	243923
14	292586
15	340957
16	375904



What function would best fit the data (linear, exponential, or power function?)? Why?

Despite the fact that all three functions fit the data reasonably well up to 1996, it is important to realize that they give wildly different predictions for the future. If we use each model to estimate the total number of AIDS deaths by the year 2010 (when t = 30),

- the exponential model gives $N = 630e^{(0.47)30} \approx 837{,}322{,}467$, about triple the current US population;
- the power model gives $N = 107(30)^{3.005} \approx 2{,}938{,}550$, or about 1% of the current population;
- and the linear model gives $N = -97311 + 25946 \cdot 30 = 681{,}069$, or about 0.22% of the current population.

Ex: David has available 400 yards of fencing and wishes to enclose a rectangular area.

a. Express the area A of the rectangle as a function of x, where x is the length of the rectangle.

$P = 2l + 2w$
$400 = 2x + 2w$
$400 - 2x = 2w$
$200 - x = w$

$A = l \cdot w$
$A = x(200 - x)$ or $A = 200x - x^2$

b. For what value of x is the area the largest? At vertex: $x = \frac{-b}{2a} = \frac{-200}{2(-1)} = \frac{-200}{-2} = 100$

c. What is the maximum area?

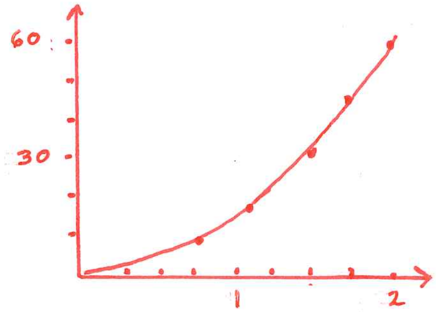
$A = 200(100) - 100^2$
$= 20000 - 10{,}000$
$A = 10{,}000\ yd^2$

or $A = 100(200 - 100)$
$100(100)$
$A = 10{,}000\ yd^2$

Ex: Paul, David's friend, doesn't believe David's estimate of acceleration due to gravity is correct. Paul repeats the experiment and obtains the following data:

Time (seconds)	Distance (feet)
.7907	10
1.1160	20
1.4760	35
1.6780	45
1.9380	60

a. Draw a scatter diagram using time as the independent variable and distance as the dependent variable.



b. Using a graphing calculator, find the power function of best fit for the data.

$y = 16.023x^{1.998}$

STAT-CALC - Pwr Reg - L_1, L_2, Y_1

c. Graph the function found in part *b* on the scatter diagram.

STAT-PLOT → 1st GRAPH

d. Use the function found in part *b* to predict the time it will take an object to fall 100 feet.

$$100 = 16.023X^{1.998}$$

$$\frac{100}{16.023} = X^{1.998}$$

$$\left(\frac{100}{16.023}\right)^{\frac{1}{1.998}} = X$$

$$2.500 = X$$

$$\boxed{X \approx 2.5 \text{ seconds}}$$

Ex: For the polynomial $f(x) = x^2(x-2)(x+2)$ find the following:

a. Find the x intercepts of the graph.

$0 = x^2(x-2)(x+2)$

$x^2 = 0 \quad x - 2 = 0 \quad x + 2 = 0$ $\qquad (0,0)\ (2,0)(-2,0)$

$x = 0 \quad x = 2 \quad x = -2$

b. Find the y intercepts of the graph.

$f(0) = 0^2(0-2)(0+2)$

$f(0) = 0 \qquad (0,0)$

c. Determine whether the graph crosses or touches the x-axis at each x-intercept.

Touches at $x=0$ Crosses at $x = 2$ $x = -2$

d. End behavior—find the power function that the graph of f resembles for large values of x.

$y = x^4$ As $x \to \infty$ $y \to \infty$ or $\lim\limits_{x \to \infty} x^4 = \infty$

$x \to -\infty$ $y \to \infty$

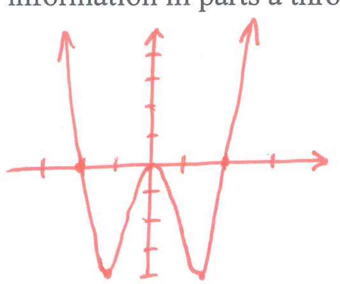
$\lim\limits_{x \to -\infty} x^4 = -\infty$

e. Use a graphing calculator to graph f.

f. Determine the number of turning points on the graph. Approximate the turning points to the nearest hundredth.

Turning points: $(-1.41, -4)$ $(0,0)$ $(1.41, -4)$

g. Use the information in parts a through f to graph f by hand.



Ex: Beth is interested in finding a function that explains the closing price of Harley Davidson stock at the end of each month. She obtains the following data:

Year (x)	**Closing Price (y)**
1987 ($x = 1$)	.392
1988	.7642
1989	1.1835
1990	1.1609
1991	2.6988
1992	4.5381
1993	5.3379
1994	6.8032
1995	7.0328
1996	11.5585
1997	13.4799
1998	23.5424
1999	31.9342
2000	39.7277

a. Using a graphing calculator, draw a scatter diagram with year as the independent variable.

b. Using a graphing calculator, fit an exponential function to the data.

STAT → CALC → EXP REG L1, L2, Y1

$y = .3944(1.4028)^x$

c. Express the function found in part b in the form $A = A_0 e^{kt}$.

$A = .39437e^{.33847t}$

$.394(1.4028)^x = A_0 e^{kt}$

$.394(1.4028)^x = .394(e^k)^t$

$1.4028^x = (e^k)^t$

$1.4028 = e^k$

$1.4028 = e^k$

$\ln 1.4028 = \ln(e^k)$

$\ln 1.4028 = k$

$.33847 = k$

d. Graph the exponential function found in part b or c on the scatter diagram.

e. Using the solution to part b or c, predict the closing price of Harley Davidson stock at the end of 2001.

$A = .39437e^{(.33847)(15)}$ or $y = .3944(1.4028)^{15}$ Graph = \$63.22

$A = 62.22$ $y = \$63.22$

f. Interpret the value of k found in part c.

Continuous growth rate of 33.8%

HW: pg 477 – 481 #2,3,5,13,18,20,23,31